Common Points on Elliptic Curves:

The Achilles' Heel of Fault Attack Countermeasures

Alberto Battistello

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Overview

1. Introduction
2. A Simple Fault Attack
3. Our New Attack
4. Conclusion

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Overview

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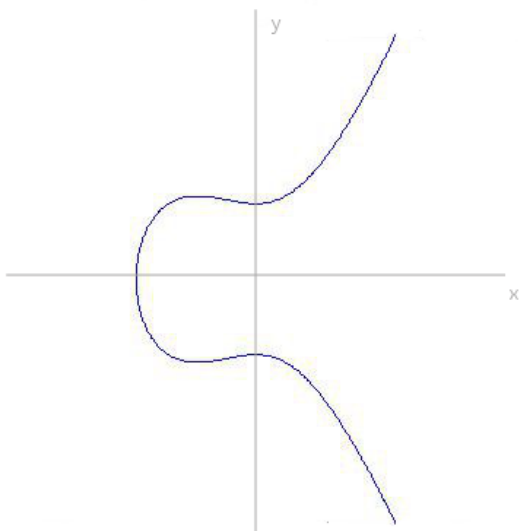
Short Weierstrass Equation

Curve Equation

$$\mathcal{E}(\mathbb{F}_p) : y^2 = x^3 + \mathbf{a} \cdot x + \mathbf{b} \bmod p$$

$$P = (x_p, y_p)$$



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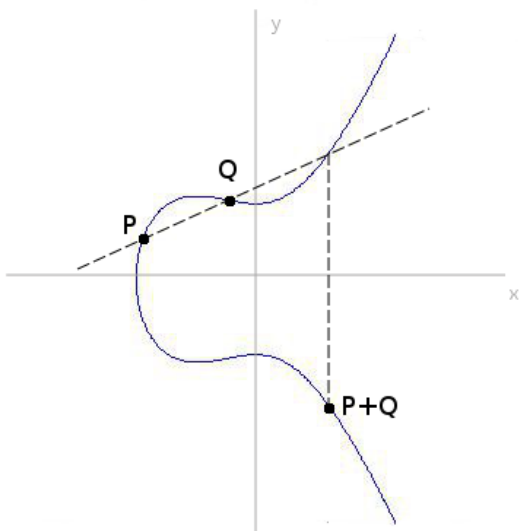
Addition/Doubling

- $P + Q = f_A(P, Q, \mathcal{E})$
- $P + P = f_D(P, \mathcal{E})$

b is never used.



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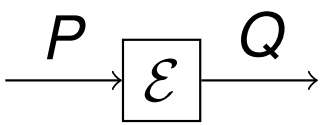
Scalar Multiplication

$$\mathcal{E}(\mathbb{F}_p) : y^2 = x^3 + a \cdot x + b \bmod p$$

Scalar Multiplication

$$[d]P = \underbrace{P + P + P + P}_{\text{d times}} = Q$$



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ECDLP

ECDLP

Given $P, Q \in \mathcal{E}(\mathbb{F}_p)$ find d such that $Q = [d]P$.
The ECDLP complexity depends on the order of P over $\mathcal{E}(\mathbb{F}_p)$.

Order of Point

$Ord_{\mathcal{E}}(P) = \prod q_i^{\alpha_i}$.

max $q_i \leq$ 112-bit $\Rightarrow$ ρ-Pollard.

The same point on another curve may have a weaker order.

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$\boxed{\mathcal{E}}$

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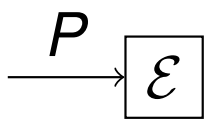

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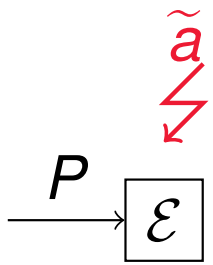

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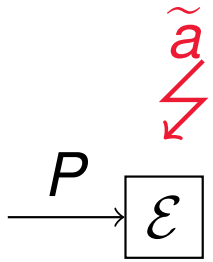

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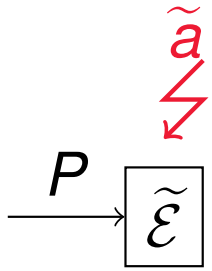

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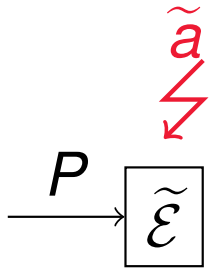

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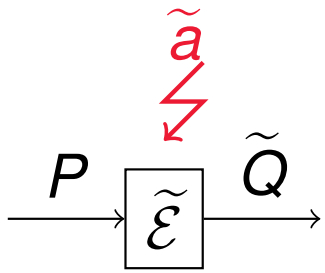

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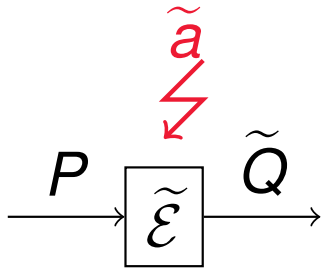
6. $\widetilde{Q} \Rightarrow d$



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Two simple countermeasures

Parameters checksums

Checksum of all parameters $(a, b, p, x_p, y_p, \ldots)$ checked before output.

$CRC(\widetilde{a}) \neq CRC(a)$

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Two simple countermeasures

Parameters checksums

Checksum of all parameters $(a, b, p, x_p, y_p, \ldots)$ checked before output.

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Point on curve test

- Initial test $y_p^2 = x_p^3 + a \cdot x_p + b \bmod p$
- Final test $y_q^2 = x_q^3 + a \cdot x_q + b \bmod p$

$y_q^2 \neq x_q^3 + \widetilde{a} \cdot x_q + b \bmod p$

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Question

The two tests have been considered equivalent . . .

up to NOW

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Common Points

$$y^2 = x^3 + a \cdot x + b \bmod p$$

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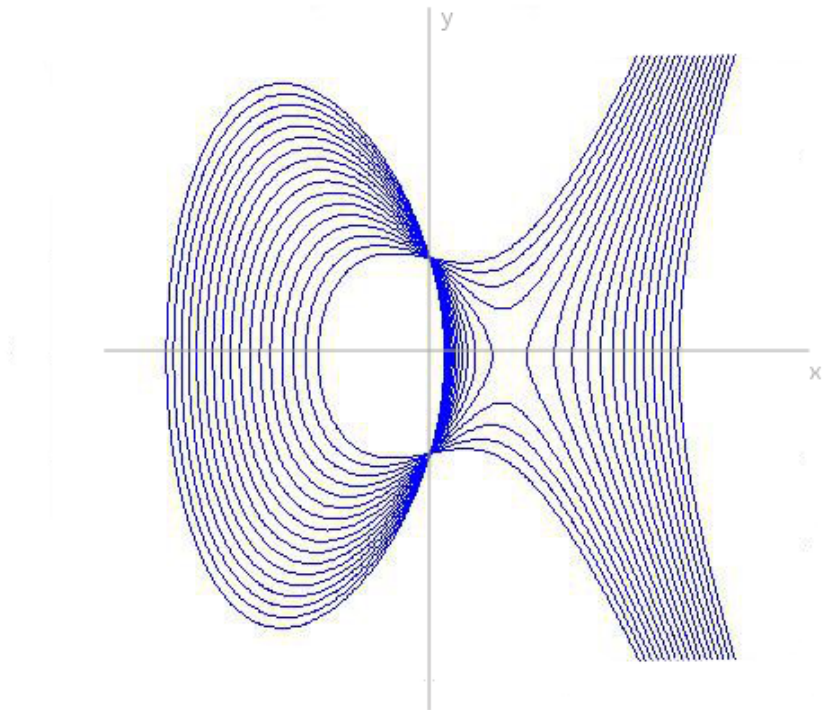

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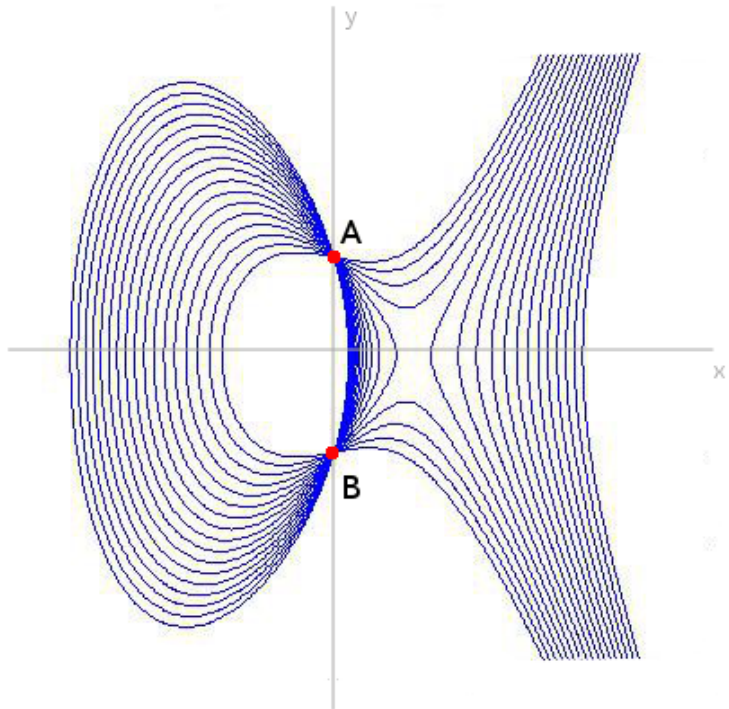
Common Points:

$$A, B : (0, \pm\sqrt{b})$$



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Attack PoC tests

$$\mathcal{E}(\mathbb{F}_p) : y^2 = x^3 + a \cdot x + b \bmod p$$

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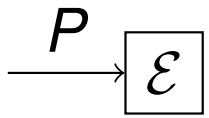

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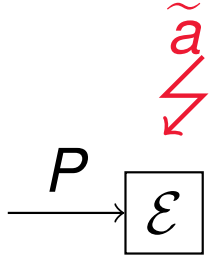

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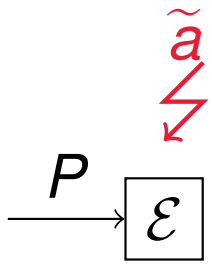

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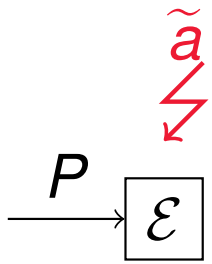

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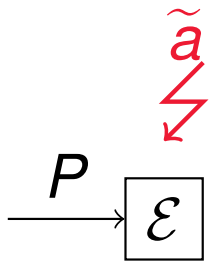

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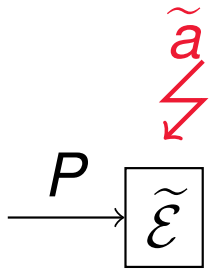

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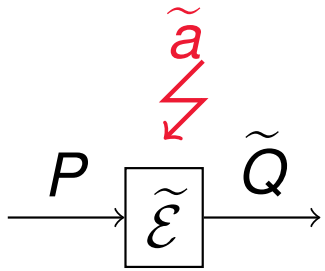

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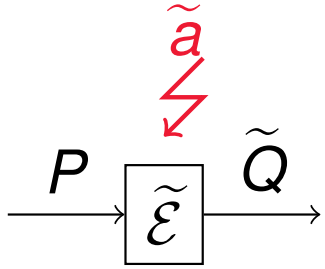

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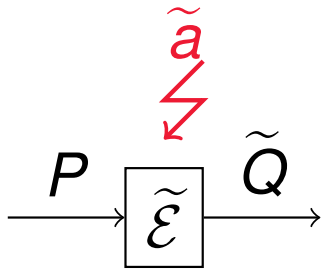

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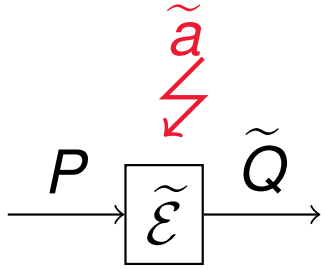
6. $Ord_{\widetilde{\mathcal{E}}}(P) \neq Ord_{\mathcal{E}}(P)$
7. $\widetilde{Q} = [d]P \Leftarrow$ **ECDL**



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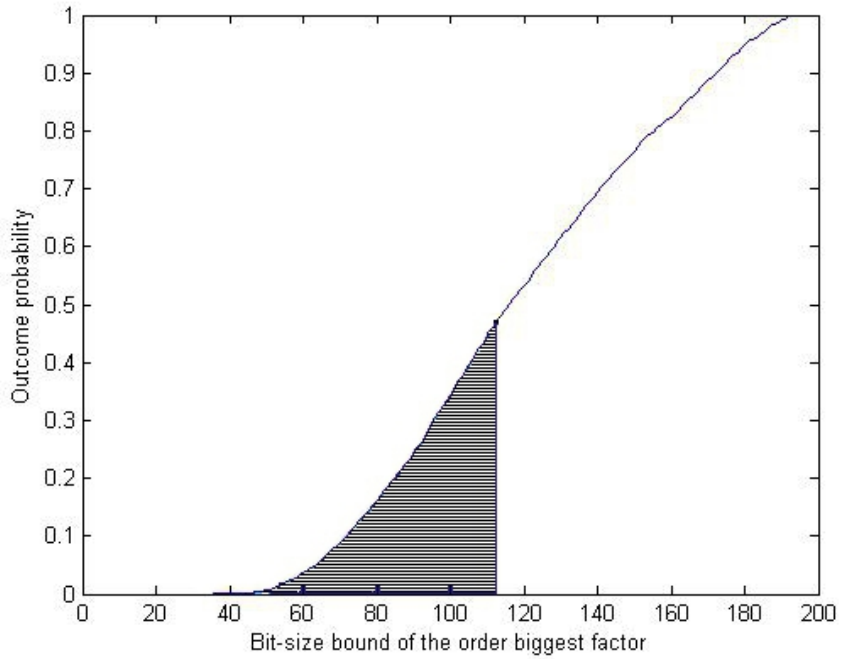
Simulations

Faults producing a weak curve.



Probability that the biggest factor is smaller than ECDLP record of 112 bit is more than 45%.

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Overview

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Conclusion

Common Points

Common Points have never been remarked before.

PoC vs Checksums

The two countermeasures are NOT equivalent!

Curves

Parameter *b* must be a square.

- NIST: P-192, P-256, P-384.
- Secp: 192r1, 256r1, 384r1, 521r1.

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Questions?

Thank you

Feel free to ask any question